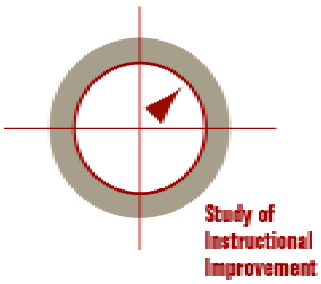

APPENDIX A:

*Measuring Teachers' Pedagogical Content Knowledge in Surveys: Detailed Results for the Domain of Mathematics**

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Appendix A:
Measuring Teachers' Pedagogical Content Knowledge in Surveys: Detailed Results for the Curricular Domain of Mathematics

This appendix presents a detailed analysis of the measures of pedagogical content knowledge developed in the area of mathematics.

Item Pool

As Table 1 shows, we began work in the curricular domain of mathematics with a variety of scenarios, each of which contained multiple items.[1] As Table 1 shows, we developed 13 different scenarios to measure the facet of pedagogical content knowledge that we called "content knowledge," and within these scenarios, we had available for measurement analysis a total of 32 separate items. For the facet labeled "knowledge of students' thinking," we developed 9 different scenarios, with a total of 39 items.

Table 1: Number of Items Assessing Teachers' Pedagogical Content Knowledge in Mathematics

	Facet of Pedagogical Content Knowledge	
	Content Knowledge	**Knowledge of Students' Thinking**
• **Number Concepts**	**10 items (3 scenarios)**	**4 items (1 scenario)**
• **Place Value**	**4 items (4 scenarios)**	**19 items (4 scenarios)**
• **Operations**	**1 item (1 scenario)**	**4 items (1 scenario)**
• **Multi-digit Computation**	**17 items (5 scenarios)**	**12 items (3 scenarios)**

Results

In the following pages, we first present scales constructed at the most fine-grained level of analysis, where we are measuring a single facet of teachers' pedagogical content knowledge in a single, "fine-grained" area of the mathematics curriculum. We then present scales at larger grain sizes, that is: (a) scales that seek to measure both facets of pedagogical content knowledge in a single, fine-grained domain; and (b) scales that seek to measure a single facet of teachers' pedagogical content knowledge across all of the fine-grained curricular areas sampled here.

Given the format of the questionnaire items used in the study, it is not possible to show the scenarios and items used to form particular scales within the body of the tables. Instead, in each table, we simply list a number for each item (e.g., B22a) and provide a very brief description of item content (in both the text and the table). Please note that the item

[1] In this paper, we define an "item" as any response option embedded within a multiple choice question that was scored as "correct" or "incorrect" for measurement purposes. Using this approach, for example, Figure 1 above includes one scenario, a single multiple choice question, and seven items; Figure 2 above includes one scenario, a single multiple choice question, and one item.

numbers listed in the tables refer to the questionnaire form (A or B), as well as the scenario/items from which the scale was built. Thus, the referent "B22a" refers to scenario 22, item a, from form B. Readers interested in examining the exact wording and format of the items included in scales are referred to the questionnaires attached to this report.

The relevant results are presented in Tables 2 – 13. Each of the tables shows all of the items considered for inclusion into a scale, and each table sorts these items according to whether they were kept in or deleted from the scale. Items kept in a scale are listed in order of difficulty (as estimated by the Rasch model), with the hardest items at the top and easier items listed in descending order of difficulty. The reader will note that these estimated difficulties do not necessarily correspond to actual p-values, which are the percentage of respondents in the sample answering that item correctly. Also note that the tables include information on items that we deleted from the final scales. These items are listed in reverse order of deletion, with those deleted at later points in the analysis listed before those deleted at earlier points. Each table also shows the item-to-scale biserial correlation for all items. For deleted items, please note that the biserial correlation listed is the one estimated at the stage just prior to deletion. For retained items, the biserial correlation is the one estimated for the final scale.

Number Concepts

Table 2 shows that we created three scales in the "fine-grained" curricular domain that we labeled number concepts. One scale was designed to measure teachers' content knowledge in this area, another scale was designed to measure teachers' knowledge of student thinking in this area, and a final scale was designed to measure both these elements of teachers' pedagogical content knowledge in this area.

Table 2 (next page) shows the scale for teachers' *content knowledge* in this domain. We began constructing this scale with a pool of 10 items, most of which were based on a single scenario (A22) asking teachers a series of questions about properties of the number zero. However, two other items in this pool asked teachers about properties of decimals. As the table shows, three items about the number zero (A22A, A22B, A22G) and a single item about decimals (B21) were retained in the final scale, and these items had biserial correlations ranging from 0.380 to 0.669. Other items on properties of the number zero (A22C through A22F, A22H) and a single item on decimals (B34) were deleted from the scale during analysis. The final, 10-item scale for teachers' content knowledge in the area of number concepts had a reliability of 0.674.

Table 3 (next page) shows the scale for teachers' *knowledge of students' thinking* in the area of number concepts. This scale consists of items from a single scenario (A29) asking respondents to explain why a group of kindergartners who are using blocks in counting would have arranged the blocks as the scenario depicts. The final scale consists of 3 out of 4 of items from this scenario and yields a final scale with the relatively low reliability of 0.52. The low reliability obtained here is at least partly a function of the small number of items used to construct the scale, but the low biserial correlations also suggest little internal consistency across teachers' responses to this scenario.

Table 2: Number Concepts Scale--Teachers' Content Knowledge			
	Reliability = 0.674		
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A22A	**0.500**	**0.634**	• **0 is an even number**
A22B	**0.603**	**0.669**	• **0 is not really a number, just a placeholder for really big numbers**
A22G	**0.898**	**0.380**	• **Multiplying by 0 gives same number**
B21	**0.735**	**0.476**	• **Which decimal is largest?**
Deleted			
A22C	**0.707**	**-0.012**	• **Number 8 can be written 008**
A22D	**0.860**	**-0.004**	• **0 is smallest number**
A22E	**0.655**	**0.041**	• **You can't subtract from 0**
A22F	**0.983**	**-0.147**	• **Adding zero to a number gives same number**
A22H	**0.448**	**0.090**	• **Dividing by 0 gives 0**
B34	**0.104**	**0.092**	• **Order a set of decimals**

Table 3: Number Concepts Scale—Knowledge of Students' Thinking			
	Reliability = 0.522		
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A29A	**.517**	**.411**	• **Students forgot how many blocks counted previously**
A29B	**.190**	**.438**	• **Students have to count blocks from start to order numbers**
A29C	**.914**	**.386**	• **Students cannot "count on" yet**
Deleted			
A22D	**.759**	**.180**	• **Counting above 5 is difficult for students at this age**

Table 4 presents a scale built from a pool of all of the items previously considered. Conceptually, we can view this as a scale measuring the larger construct of teachers' *pedagogical content knowledge*, but one that does so within the fine-grained curriculum area of number concepts. Here we are using the 14 items drawn from the 4 different scenarios discussed in the previous paragraphs. Table 4 shows that this combined scale includes items from three separate scenarios (A22, B21, A29) that include questions related to the properties of zero, ordering decimals, and counting numbers. However, the final 8-item scale has only a moderate reliability of 0.500.

Table 4: Number Concepts—Pedagogical Content Knowledge			
		Reliability = 0.500	
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A22A	**0.500**	**0.258**	• **0 is an even number**
A22B	**0.603**	**0.570**	• **0 is not really a number, just a placeholder for really big numbers**
A22E	**0.655**	**0.196**	• **You can't subtract from 0**
B21	**0.808**	**0.296**	• **Which decimal is largest?**
A29A	**0.517**	**0.217**	• **Students forgot how many blocks counted previously**
A29B	**0.190**	**0.495**	• **Students have to count blocks from start to order numbers**
A29C	**0.915**	**0.182**	• **Students cannot "count on" yet**
A29D	**0.759**	**0.327**	• **Counting above 5 is difficult for students at this age**
Deleted			
A22C	**0.707**	**0.067**	• **Number 8 can be written 008**
A22D	**0.860**	**0.073**	• **0 is smallest number**
A22F	**0.983**	**-0.044**	• **Adding zero to a number gives same number**
A22G	**0.898**	**0.127**	• **Multiplying by 0 gives same number**
A22H	**0.448**	**0.096**	• **Dividing by 0 gives 0**
B34	**0.104**	**-0.293**	• **Order a set of decimals**

Place Value

Next, we describe the results for measures of teachers' pedagogical content knowledge in the domain of place value. The measure of teachers' *content knowledge* in this area is shown in Table 5 (next page). The items available for use in this scale came from 4 different scenarios—two of which asked teachers to construct a number from bundles of sticks, where each stick had a value of one (A19, B20), and two scenarios that asked teachers how many different ways the number 23 can be represented in a base 10 system (A23) and a base six system (A24). The final results show that the four items drawn from these scenarios produced a scale with a reliability of 0.00(!), suggesting absolutely no internal consistency in teachers' responses across the items we used to measure teachers' content knowledge in this area.

Efforts to build a scale measuring teachers' *knowledge of students' thinking* in the area of place value were much more successful, as Table 6 (next page) shows. Here, we began with 19 items from 4 different scenarios, keeping 13 items and deleting another 6 items. The scenarios used in this analysis asked teachers' whether a group of students working with a particular configuration of base 10 blocks could be likely to provide particular answers to a question posed by the teacher (A25, B27), to answer whether or not first graders might make particular kinds of errors when representing particular numbers (A28), and to answer a series of questions about why a group of students representing a set of numbers in powers of ten were arriving at the answer provided in the scenario (B29). Overall, the final 13-item scale in this area had a reliability of 0.764.

Table 5: Place Value – Teachers' Content Knowledge			
	Reliability = 0.000		
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A19	**0.898**	**-0.158**	• **How many counting sticks are represented in the graph?**
A23	**0.559**	**-0.100**	• **The number of possible ways to write 72**
A24	**0.158**	**-0.244**	• **Write 72 in base 6**
B20	**0.923**	**-0.421**	• **How many counting sticks are represented in the graph?**

Table 6: Place Value – Knowledge of Students' Thinking			
	Reliability = 0.764		
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A25A	**0.911**	**0.463**	• **Possible answer from second graders: 1.5**
A25B	**0.328**	**0.604**	• **Possible answer from second graders: 150**
A25D	**0.831**	**0.306**	• **Possible answer from second graders: 105**
A25F	**0.508**	**0.478**	• **Possible answer from second graders: 15**
A28B	**0.917**	**0.340**	• **Students mix up 23 with 32**
A28C	**0.733**	**0.542**	• **Students have trouble reading two-digit numbers**
A28D	**0.583**	**0.407**	• **Students have problems counting objects accurately up to 50**
A28E	**0.867**	**0.413**	• **Students having trouble writing numerals correctly**
B27B	**0.34**	**0.608**	• **Teacher believes base ten blocks are not suited for teaching decimals**
B27C	**0.878**	**0.484**	• **Teacher answers that .35 is greater than .4**
B27D	**0.612**	**0.421**	• **Students are using blocks correctly but misinter-preting what they see**
B29A	**0.429**	**0.678**	• **Students don't know how to multiply numbers raised to the power ten correctly**
B29D	**0.24**	**0.853**	• **Teacher believes the assigned problems are too hard for the students**
Deleted			
A25C	**0.263**	**0.340**	• **Possible answer from second graders: 1005**
A25E	**0.327**	**0.051**	• **Possible answer from second graders: 6**
A28A	**0.254**	**-0.010**	• **Students write 23 as 203**
B27A	**0.938**	**0.001**	• **35 is greater than 4 so students assume .35 is greater than .4**
B29B	**0.694**	**0.175**	• **Students use format of expanding a number with powers of ten, but don't understand what it means**
B29C	**0.102**	**0.047**	• **Students don't know what powers of ten are, what they mean, or how they work**

Table 7 shows the scale we built by combining all of the place value items to form a single scale measuring teachers' overall *pedagogical content knowledge* in this fine-grained area of the curriculum. Here, the 22 items from the 7 different scenarios previously discussed produced a final scale composed of 14 items from 3 different scenarios. Surprisingly, all of the items retained in the scale come from scenarios intended to assess teachers' knowledge of student thinking (A25, A28, B27), and not surprisingly, the resulting scale has a great deal of item overlap with the knowledge of student thinking scale discussed in the previous paragraph, as well as a reliability of 0.767—remarkably close to the reliability of the similar scale.

Table 7: Place Value – Pedagogical Content Knowledge

Reliability = 0.767

Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A25A	**0.909**	**0.463**	• **Possible answer from second graders: 1.5**
A25B	**0.333**	**0.604**	• **Possible answer from second graders: 150**
A25C	**0.263**	**0.306**	• **Possible answer from second graders: 1005**
A25D	**0.828**	**0.478**	• **Possible answer from second graders: 105**
A25F	**0.500**	**0.340**	• **Possible answer from second graders: 15**
A28B	**0.915**	**0.542**	• **Students mix up 23 with 32**
A28C	**0.729**	**0.407**	• **Students have trouble reading two-digit numbers**
A28D	**0.576**	**0.413**	• **Students have trouble counting accurately up to 50**
A28E	**0.864**	**0.608**	• **Students having trouble writing numerals correctly**
B27B	**0.327**	**0.484**	• **Teacher believes base ten blocks are not suited for teaching decimals**
B27C	**0.875**	**0.421**	• **Teacher states that .35 is greater than .4**
B27D	**0.604**	**0.678**	• **Students are not interpreting blocks correctly**
B29A	**0.412**	**0.853**	• **Students do no know how to raise numbers raised to power of ten correctly**
B29D	**0.224**	**0.094**	• **Teacher believes two-digit examples of powers of ten are more appropriate**
Deleted			
A19	**0.898**	**0.094**	• **How many counting sticks are represented in the graph?**
A23	**0.559**	**0.223**	• **The number of possible ways to write 72**
A24	**0.158**	**0.067**	• **Write 72 in base 6**
B20	**0.880**	**0.145**	• **How many counting sticks are represented in the graph?**
A25E	**0.327**	**0.123**	• **Possible answer from second graders: 6**
A28A	**0.254**	**0.038**	• **Students write 23 as 203**
B27A	**0.938**	**-0.110**	• **35 is greater than 4 so students assume .35 is greater than .4**
B29B	**0.694**	**0.172**	• **Students use format of expanding a number with powers of ten, but don't understand what it means**
B29C	**0.102**	**0.150**	• **Students do not understand powers of ten**

Operations. We had only a single item (A20) measuring teachers' *content knowledge* in the area of operations and therefore could not construct a scale in this area. Four items were available for teachers' *knowledge of students' thinking* in this area. However, all of these items came from a single scenario (A27) asking teachers to tell whether a variety of possible answers to a word problem involving a difference in quantities were acceptable mathematically. As Table 8 shows, one of the four available items from this scenario was dropped from the scale, and the remaining three items formed a scale with a reliability of 0.545.

Table 8: Operations –Knowledge of Students' Thinking

Reliability = 0.545			
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A20	**0.362**	**0.287**	• **Teacher asked to select correct number sentence corresponding to graph**
A27B	**0.797**	**0.407**	• **Teacher asked to determine whether a particular mathematical method for a solution is acceptable: addition**
A27C	**0.914**	**0.753**	• **Teacher asked to determine whether a particular mathematical method for a solution is acceptable: counting up**
A27D	**0.712**	**0.595**	• **Teacher asked to determine whether a particular mathematical method for a solution is acceptable: counting sticks**
Deleted			
B22	**0.077**	**-0.077**	• **What power of 10 equals 1?**
A27A	**1.000**	**NA**	• **Teacher asked to determine whether a particular mathematical method for a solution is acceptable: subtraction**

Multi-digit Computation

Questionnaire items on operations designed for upper grade teachers focused on scenarios related to multi-digit computation. Table 9 (next page) shows the measure of teachers' *content knowledge* in this area. Here, we had 17 items available from 5 different scenarios. The scenarios typically asked teachers to decide if a particular procedure for performing operations would work under varying circumstances, where the operations considered were multi-digit subtraction (A21, B25), multi-digit multiplication (B23), and multiplication of decimals (B24). Of the 17 available items, we retained 14 in the final scale, obtaining a reliability of 0.859. As the data in the table show, the scale obtains reliability not simply because it includes a large number of items, although this is a factor contributing to higher reliability, but also because all of the retained items had item-to-scale correlations in the 0.50 to 0.90 range.

Table 9: Multi-digit Computation – Teachers' Content Knowledge			
	Reliability = 0.859		
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A21	**0.320**	**0.521**	• **Teacher's knowledge of subtraction methods**
B23B	**0.898**	**0.645**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B23C	**0.673**	**0.529**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B23D	**0.240**	**0.676**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B24A	**0.438**	**0.629**	• **Teacher's knowledge of multiplication procedures with decimals**
B24B	**0.771**	**0.908**	• **Teacher's knowledge of multiplication procedures with decimals**
B24C	**0.510**	**0.653**	• **Teacher's knowledge of multiplication procedures with decimals**
B24D	**0.130**	**0.687**	• **Teacher's knowledge of multiplication procedures with decimals**
B25A	**0.940**	**0.667**	• **Teacher's knowledge of mathematically acceptable solutions**
B25C	**0.860**	**0.575**	• **Teacher's knowledge of mathematically acceptable solutions**
B26A	**0.880**	**0.567**	• **Teacher's knowledge of mathematically acceptable solutions**
B26B	**0.396**	**0.611**	• **Teacher's knowledge of mathematically acceptable solutions**
B26C	**0.673**	**0.802**	• **Teacher's knowledge of mathematically acceptable solutions**
B26D	**0.327**	**0.618**	• **Teacher's knowledge of mathematically acceptable solutions**
Deleted			
B23A	**0.638**	**0.212**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B25B	**0.490**	**0.200**	• **Teacher's knowledge of mathematically acceptable solutions**
B25D	**0.417**	**0.312**	• **Teacher's knowledge of mathematically acceptable solutions**

Fewer items were available to measure teachers' *knowledge of student thinking* in the area of multi-digit computation, but as Table 10 (next page) shows, we still achieved a scale with a reliability of 0.744 using only 5 of the 12 items initially available for scaling. Of the items retained in the scale, two came from a scenario (A26) asking teachers to evaluate a series of explanations for why a student might have gotten the answer she did to a subtraction problem, while the remaining three items in the scale came from a scenario (B30) asking teachers' to evaluate a series of statements that conjecture about how a student is thinking about a multiplication problem given that student's response to the problem as described in the scenario. None of the items from a third scenario (B28) asking teachers to explain why a stu-

dent might have arrived at the answer to a multi-digit subtraction problem shown in the scenario were included in the final scale.

Table 10: Multi-digit Computation – Knowledge of Students' Thinking			
Reliability = 0.744			
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A26B	**0.732**	**0.826**	• **Student does not know basic subtraction facts**
A26C	**0.931**	**0.999**	• **Student subtracted instead of regrouping**
B30A	**0.875**	**0.860**	• **Students potential use/misuse of mathematical terminology**
B30B	**0.875**	**0.531**	• **Student's thinking/method does not work equally across all types of operations**
B30D	**0.673**	**0.496**	• **Teacher unsure of student's thinking/method**
Deleted			
A26A	**0.386**	**-0.068**	• **Student forgot to cross out when regrouping**
A26D	**0.474**	**-0.045**	• **Student does not understand place value**
B28A	**0.833**	**0.057**	• **Student has difficulty with subtraction facts**
B28B	**0.750**	**-0.199**	• **Student subtracted instead of regrouping**
B28C	**0.880**	**-0.271**	• **Student forgot to cross out when regrouping**
B28D	**0.667**	**0.027**	• **Student does not understand place value**
B30C	**0.660**	**0.141**	• **Students potential use/misuse of mathematical terminology**

We also attempted to construct a Multi-digit Computation *pedagogical content knowledge* scale by combining the items from both the *content knowledge* and *knowledge of students' thinking* scales presented above. A total of 29 items were entered into the analysis of which 20 were retained in the final scale. Table 11 (next page) shows the reliability for this combined scale to be 0.875, which is only a slight improvement over the reliability of 0.859 for the content knowledge scale alone. Given the additional items introduced to this scale, the Spearman-Brown prophecy formula predicts an increase in reliability to at least 0.900, however, providing at least some evidence of a lack of unidimensionality in the scale. Moreover, the measure order statistics show that both the content knowledge and knowledge of students' thinking measures tend to group along their respective dimensions.

Table 11: Multi-digit Computation – All Items			
Reliability = 0.874			
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
B23B	**0.898**	**0.960**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B23C	**0.673**	**0.669**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B23D	**0.240**	**0.671**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B24A	**0.438**	**0.541**	• **Teacher's knowledge of multiplication procedures with decimals**
B24B	**0.771**	**0.944**	• **Teacher's knowledge of multiplication procedures with decimals**
B24C	**0.510**	**0.682**	• **Teacher's knowledge of multiplication procedures with decimals**
B24D	**0.130**	**0.642**	• **Teacher's knowledge of multiplication procedures with decimals**
B25A	**0.040**	**0.563**	• **Teacher's knowledge of mathematically acceptable solutions**
B25C	**0.860**	**0.604**	• **Teacher's knowledge of mathematically acceptable solutions**
B25D	**0.417**	**0.331**	• **Teacher's knowledge of mathematically acceptable solutions**
B26A	**0.880**	**0.600**	• **Teacher's knowledge of mathematically acceptable solutions**
B26B	**0.396**	**0.504**	• **Teacher's knowledge of mathematically acceptable solutions**
B26C	**0.673**	**0.761**	• **Teacher's knowledge of mathematically acceptable solutions**
B26D	**0.327**	**0.578**	• **Teacher's knowledge of mathematically acceptable solutions**
A26B	**0.732**	**0.799**	• **Teacher's knowledge of mathematically acceptable solutions**
A26C	**0.931**	**0.999**	• **Teacher's knowledge of mathematically acceptable solutions**
B28B	**0.750**	**0.410**	• **Student subtracted instead of regrouping**
B30A	**0.875**	**0.859**	• **Students potential use/misuse of mathematical terminology**
B30B	**0.875**	**0.665**	• **Student's thinking/method does not work equally across all types of operations**
B30D	**0.660**	**0.463**	• **Teacher unsure of student's thinking/method**
Table continues on next page with deleted items			

Table 11: Multi-digit Computation – All Items (continued)			
Deleted			
A21	**0.259**	**0.173**	• **Teacher's knowledge of subtraction methods**
B23A	**0.638**	**0.114**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B25B	**0.490**	**0.121**	• **Teacher's knowledge of mathematically acceptable solutions**
A26A	**0.386**	**-0.032**	• **Student forgot to cross out when regrouping**
A26D	**0.474**	**0.060**	• **Student does not understand place value**
B28A	**0.833**	**0.041**	• **Student has difficulty with subtraction facts**
B28C	**0.880**	**-0.222**	• **Student forgot to cross out when regrouping**
B28D	**0.521**	**0.002**	• **Student does not understand place value**
B30C	**0.667**	**0.071**	• **Students potential use/misuse of mathematical terminology**

Measures at Larger Grain Sizes

To this point, we have discussed scale construction efforts that were limited to measuring teacher pedagogical content knowledge within a specific, fine-grained area of the curriculum. In this section, we turn to measures at larger grained sizes. In particular, we report here on two scales we constructed to represent the overall dimensions of *content knowledge* and *knowledge of students' thinking* across all of the "fine-grained" curricular topics in mathematics that we were sampling.

The results for overall c*ontent knowledge in mathematics* are presented in Table 12 (next page). Of the original 33 items put into the analysis, 23 items remained in the final scale. The final content knowledge scale consists of 14 items from the domain of multi-digit computation, 2 items each from the domains of operations and from place value scales, and 5 items from the domain of number concepts. Biserial correlations for items kept in this scale ranged from 0.218 to 0.836, and the reliability of the final measure was 0.869. It is noteworthy that the items kept from the domain of number in this measure were mostly items that had been deleted from the Number Concepts – Content Knowledge scale discussed in Table 4 above. Conversely, items that were kept in the Number Concepts – CK scale shown in Table 4 had zero or negative biserial correlations in the context of the scale discussed here. Thus it appears that for these data the Number Concepts – Content Knowledge scale described in Table 4 is measuring something idiosyncratic, which is unrelated to other aspects of Mathematical Content Knowledge.

Table 12: Teachers' Content Knowledge in Mathematics			
		Reliability = 0.869	
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A20	**0.362**	**0.362**	• **Teacher asked to select correct number sentence corresponding to graph**
A21	**0.259**	**0.427**	• **Teacher's knowledge of subtraction methods**
A22C	**0.707**	**0.412**	• **Number 8 can be written 008**
A22D	**0.860**	**0.674**	• **0 is smallest number**
A22E	**0.655**	**0.421**	• **You can't subtract from 0**
A22H	**0.448**	**0.462**	• **Dividing by 0 gives 0**
A23	**0.559**	**0.462**	• **The number of possible ways to write 72**
A24	**0.158**	**0.218**	• **Write 72 in base 6**
B22	**0.200**	**0.368**	• **What power of 10 equals 1?**
B23B	**0.898**	**0.745**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B23C	**0.673**	**0.498**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B23D	**0.240**	**0.681**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B24A	**0.438**	**0.559**	• **Teacher's knowledge of multiplication procedures with decimals**
B24B	**0.771**	**0.836**	• **Teacher's knowledge of multiplication procedures with decimals**
B24C	**0.510**	**0.611**	• **Teacher's knowledge of multiplication procedures with decimals**
B24D	**0.130**	**0.689**	• **Teacher's knowledge of multiplication procedures with decimals**
B25A	**0.940**	**0.597**	• **Teacher's knowledge of mathematically acceptable solutions**
B25C	**0.860**	**0.631**	• **Teacher's knowledge of mathematically acceptable solutions**
B25D	**0.417**	**0.386**	• **Teacher's knowledge of mathematically acceptable solutions**
B26A	**0.880**	**0.550**	• **Teacher's knowledge of mathematically acceptable solutions**
B26B	**0.396**	**0.583**	• **Teacher's knowledge of mathematically acceptable solutions**
B26C	**0.673**	**0.741**	• **Teacher's knowledge of mathematically acceptable solutions**
B26D	**0.327**	**0.718**	• **Teacher's knowledge of mathematically acceptable solutions**
B34	**0.104**	**0.484**	• **Order a set of decimals**
Table continues on next page with deleted items			

Table 12 (cont.): Teachers' Content Knowledge in Mathematics			
Deleted			
A19	**0.898**	**-0.186**	• **How many counting sticks are represented in the graph?**
A22A	**0.500**	**-0.200**	• **0 is an even number**
A22B	**0.603**	**-0.130**	• **0 is not really a number, just a placeholder for really big numbers**
A22F	**0.983**	**0.075**	• **Adding zero to a number gives same number**
A22G	**0.898**	**-0.041**	• **Multiplying by 0 gives same number**
B20	**0.880**	**-0.139**	• **How many sticks are represented in the graph?**
B21	**0.740**	**-0.151**	• **Which decimal is largest?**
B23A	**0.638**	**0.191**	• **Teacher's knowledge of multiplication procedures with whole numbers**
B25B	**0.490**	**0.212**	• **Teacher's knowledge of mathematically acceptable solutions**

Results for the overall *Knowledge of Students' Thinking* scale are given in Table 13. Of the 39 available items for this dimension, 24 demonstrated acceptable scaling properties (15 items from place value, 7 items from multi-digit computation, and one each from number concepts and operations. Not surprisingly, many of the items deleted in this scale performed poorly in the "fine-grained" subscales, especially items in the topic areas of number concepts and operations. Further, the results for this dimension of teachers' knowledge were not on par with the overall Content Knowledge scale. The biserial correlations for kept items ranged from 0.195 to .0669 and the reliability of the final scale was 0.785.

Table 13: Knowledge of Students' Thinking in Mathematics			
Reliability = 0.785			
Items	***% Correct***	***Item-to-Scale Biserial***	**Item Content**
Kept			
A25A	**0.909**	**0.478**	• **Possible answer from second graders: 1.5**
A25B	**0.333**	**0.669**	• **Possible answer from second graders: 150**
A25C	**0.263**	**0.240**	• **Possible answer from second graders: 1005**
A25D	**0.828**	**0.382**	• **Possible answer from second graders: 105**
A25F	**0.500**	**0.391**	• **Possible answer from second graders: 15**
A26B	**0.732**	**0.550**	• **Student does not know basic subtraction facts**
A26C	**0.932**	**0.501**	• **Student subtracted instead of regrouping**
A26D	**0.474**	**0.371**	• **Student does not understand place value**
A27C	**0.914**	**0.326**	• **Acceptable mathematical method for solution: counting up**
A28B	**0.915**	**0.347**	• **Students mix up 23 with 32**
A28D	**0.576**	**0.290**	• **Students have problems counting objects accurately up to 50**
Table continues on next page with both kept and deleted items			

Table 13 (cont.): Knowledge of Students' Thinking in Mathematics			
A28E	**0.864**	**0.394**	• **Students having trouble writing numerals correctly**
A29D	**0.759**	**0.327**	• **Counting above 5 is difficult for students at this age**
B27B	**0.327**	**0.489**	• **Teacher believes base ten blocks are not suited for teaching decimals**
B27C	**0.875**	**0.616**	• **Teacher answers that .35 is greater than .4**
B27D	**0.604**	**0.494**	• **Students are using blocks correctly but misinterpreting what they see**
B28D	**0.521**	**0.195**	• **Student does not understand place value**
B29A	**0.412**	**0.539**	• **Students don't know how to multiply numbers raised to the power ten correctly**
B29B	**0.694**	**0.253**	• **Students use format of expanding a number with powers of ten, but don't understand what it means**
B29D	**0.224**	**0.471**	• **Teacher believes the assigned problems are too hard for the students**
B30A	**0.875**	**0.473**	• **Students potential use/misuse of mathematical terminology**
B30B	**0.875**	**0.229**	• **Student's thinking/method does not work equally across all types of operations**
B30D	**0.660**	**0.377**	• **Teacher unsure of student's thinking/method**
Deleted			
A25E	**0.327**	**0.158**	• **Possible answer from second graders: 6**
A26A	**0.386**	**0.034**	• **Student forgot to cross out when regrouping**
A27A	**1.000**	**NA**	• **Acceptable mathematical method for solution: subtraction**
A27B	**0.797**	**-0.084**	• **Acceptable mathematical method for solution: addition**
A27D	**0.712**	**0.092**	• **Acceptable mathematical method for solution: counting sticks**
A28A	**0.254**	**0.060**	• **Students write 23 as 203**
A29A	**0.517**	**-0.055**	• **Students forgot how many blocks counted previously**
A29B	**0.190**	**-0.016**	• **Students have to count blocks from start to order numbers**
A29C	**0.915**	**0.110**	• **Students cannot "count on" yet**
B27A	**0.938**	**-0.046**	• **35 is greater than 4 so students assume .35 is greater than .4**
B28A	**0.833**	**-0.247**	• **Student has difficulty with subtraction facts**
B28B	**0.750**	**-0.049**	• **Student subtracted instead of regrouping**
B28C	**0.880**	**-0.197**	• **Student forgot to cross out when regrouping**
B29C	**0.102**	**-0.037**	• **Students don't know what powers of ten are, what they mean, or how they work**
B30C	**0.667**	**0.181**	• **Students potential use/misuse of mathematical terminology**